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THE DESIGN OF BAND SEPARATION FILTERS

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THE DESIGN OF BAND SEPARATION FILTERS

by

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ABSTRACT

A band separation filter is a network with one input and m outputs, each
corresponding to a different portion of the frequency spectrum. When a voltage
is applied to the input terminal, it will appear at one of the output terminals
only slightly attenuated. The filter considered here is a lossless network with
each output terminal terminated in a one ohm resistance. The further condition
that the input impedance of this network equals $1 + j0$ for all frequencies is
imposed.

In this thesis a sufficient condition for realizability on the m transfer
impedances is derived. It is shown that Butterworth characteristics for each
of the m transfer impedances can be achieved with networks synthesizable in
ladder form. It is also shown that L filter characteristics are also realizable
but that the synthesis procedure is more complicated and necessitates coupled
coils. Normalized curves of the attenuation characteristics for each type are
presented.

The extension of this method to transmission line networks is discussed,
and it is shown that the Butterworth characteristic can be achieved with this
type of element.

Thesis Supervisor: Elie J. Baghdady
Title: Assistant Professor of Electrical Engineering

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I.   INTRODUCTION

Methods for designing filters which reject unwanted signals while pass-ing the desired ones are quite well known and many different design procedures are available.   In many applications one wants to separate signals of various frequencies and deliver them to different loads.   The desired network then has one input and m output terminals, each output terminal corresponding to a dif-ferent portion of the frequency spectrum.   A signal at the input would then ap-pear at one of these output terminals corresponding to its frequency with little attenuation and at all other output terminals greatly attenuated.   This can be achieved by designing m filters, the first with passband from zero to ω_1, the second with passband ω_1 to ω_2 and the mth with passband ω_{m-1} to ∞.   These filters can then have their inputs connected in series or in parallel to form a single input.   If we are to make efficient use of the available power, we must require that the input impedance match the source impedance at all frequencies.   We shall, therefore, require that the input impedance equal $1 + j0$ for all frequencies where the impedance has been normalized for convenience.   To minimize un-wanted loss, we shall further restrict each filter to be a lossless network terminated in a one ohm resistance.   The network will then take the form shown in Figure (1a) and (1b).

The problem then is to synthesize m networks having the correct pass-band characteristics which will have the property that either

$$Z(s) = \sum_{i=1}^{m} Z_i(s) = 1 \tag{1a}$$

or

$$Y(s) = \sum_{i=1}^{m} Y_i(s) = 1 \tag{1b}$$
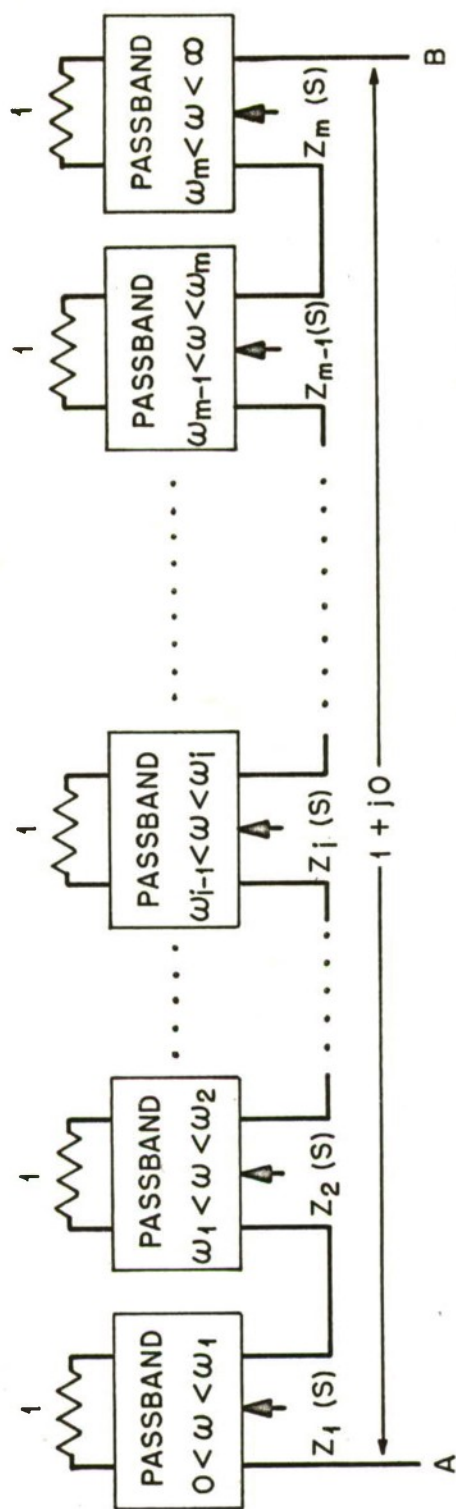
(a) SERIES CONNECTION: THE NETWORKS ARE ALL LOSSLESS

$$\sum_{i=1}^{m} Z_i(S) = 1$$
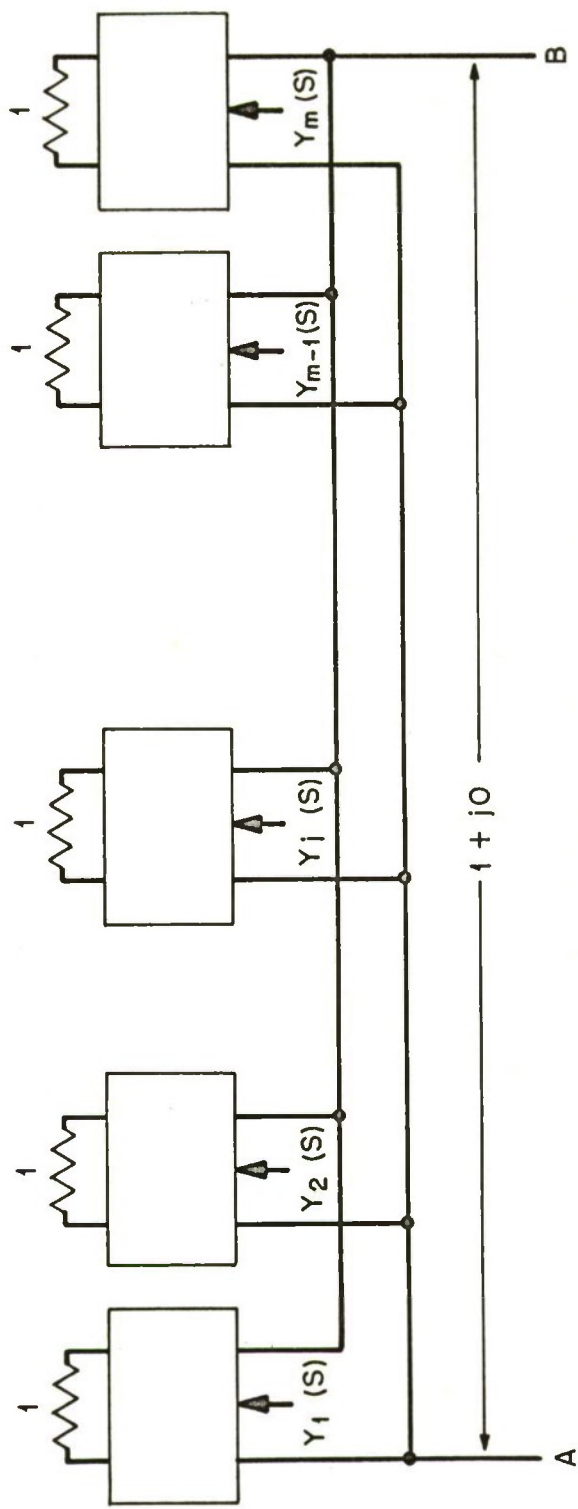
(b) PARALLEL CONNECTION: THE NETWORKS ARE LOSSLESS

$$\sum_{i=1}^{m} Y_i(S) = 1$$

FIG. 1 BLOCK DIAGRAM OF BAND SEPARATION FILTER

where $Z_i(s)$ and $Y_i(s)$ are the input impedance and admittance of the ith network. The first part of the problem is to determine which approximations to the ideal lowpass and highpass characteristic will satisfy Eq. (1a) or (1b) for realizable networks. These solutions must then be compared to see which yields the best characteristic for this specific application and which can be most easily or practicably synthesized.

II.   GENERAL PROCEDURE

Let us consider a lossless network terminated in a one ohm resistance as shown in Figure 2 with input impedance $Z(s)$ and input admittance $Y(s)$.

Let us define the transfer impedance and transfer admittance by

$$Z_T(s) = E_o(s)/I_1(s) \tag{2a}$$

$$Y_T(s) = I_o(s)/E_1(s) \tag{2b}$$

The average power delivered to the network is given by

$$P_{in} = 1/2 \left| I_1 \right|^2 \mathrm{Re}\left[Z(s) \right]_{s=j\omega} \tag{3a}$$

$$P_{in} = 1/2 \left| E_1 \right|^2 \mathrm{Re}\left[Y(s) \right]_{s=j\omega} \tag{3b}$$

Since the network is lossless, all the power is delivered to the load. Hence,

$$P_{in} = 1/2 \left| I_o \right|^2 = 1/2 \left| E_o \right|^2 \tag{4}$$

FIG. 2   A LOSSLESS NETWORK TERMINATED IN A ONE OHM RESISTANCE

Substituting Eq. (4) into Eq. (3a) and (3b):

$$|I_1|^2 \, \text{Re}\,[Z(s)]_{s=j\omega} = |E_o|^2 \tag{5a}$$

$$|E_1|^2 \, \text{Re}\,[Y(s)]_{s=j\omega} = |I_o|^2 \tag{5b}$$

and

$$\text{Re}\,[Z(s)]_{s=j\omega} = |E_o/I_1|^2 = |Z_T(j\omega)|^2 \tag{6a}$$

$$\text{Re}\,[Y(s)]_{s=j\omega} = |I_o/E_1|^2 = |Y_T(j\omega)|^2 \tag{6b}$$

The condition of Eq. (1a) and (1b) can be written as:

$$\sum_{i=1}^{m} \text{Re}\,[Z_i(s)]_{s=j\omega} = 1 \tag{7a}$$

$$\sum_{i=1}^{m} \text{Im}\,[Z_i(s)]_{s=j\omega} = 0 \tag{7b}$$

$$\sum_{i=1}^{m} \text{Re}\,[Y_i(s)]_{s=j\omega} = 1 \tag{7c}$$

$$\sum_{i=1}^{m} \text{Im}\,[Y_i(s)]_{s=j\omega} = 0 \tag{7d}$$

Using Eq. (6a) and (6b), the condition of Eq. (7a) and (7c) can be rewritten:

$$\sum_{i=1}^{m} |Z_{Ti}(j\omega)|^2 = 1 \tag{8a}$$

$$\sum_{i=1}^{m} |Y_{Ti}(j\omega)|^2 = 1 \tag{8b}$$

We shall now show that condition (7b) must be satisfied if (7a) is satisfied and each $Z_i(s)$ is a minimum reactive network and similarly that (7d) follows from Eq. (7c) for minimum susceptive networks.

Let us write:

$$Re\,[Z(s)] = 1/2\,[Z(s) + Z(-s)] \tag{9}$$

If $Z(s)$ is minimum reactive, its poles and zeros lie in the LHP and those of $Z(-s)$ in the RHP. Hence, one can construct $Z(s)$ from the $Re\,[Z(s)]$ by choosing the poles and zeros of $Re\,[Z(s)]$ in the LHP. However, for the case at hand $Re\,[Z(s)]$ is a constant (Eq. (1a) and (1b)) and, therefore, has no poles or zeros. Therefore, $Z(s)$ is a real constant and has no imaginary part.

The problem has now been simplified since we need only consider solutions to Eq. (8a) or (8b). If Eq. (8a) is satisfied and a voltage generator with voltage 2E and an internal impedance of 1 ohm is connected across terminals AB of Figure 1a, then the input current I to each network equals E. The available input power to the network P_{in} is, therefore, equal to $|I|^2$. The output voltage of the ith network E_{oi} is equal to $I(s)Z_{Ti}(s)$ and the output power of the ith network is:

$$P_{oi} = |I|^2 |Z_{Ti}(j\omega)|^2 \tag{10}$$

Therefore,

$$P_{oi}/P_{in} = |Z_{Ti}(j\omega)|^2 = |E_{oi}/E|^2 \tag{11}$$

Similarly, if one connects the voltage source across terminals AB of Figure 1b, the available input power is $|E|^2$. The output power of the ith network is $|I_{oi}|^2 = |E_{oi}|^2$. Thus,

$$P_{oi}/P_{in} = |Y_{Ti}(j\omega)|^2 = |E_{oi}/E|^2 \tag{12}$$

Since $|E_{oi}/E|^2$ is the quantity we wish to control as a function of frequency, we need merely choose the $Z_{Ti}(s)$ or $Y_{Ti}(s)$ to have desirable passband characteristics and to correspond to realizable networks while satisfying Eq. (8a) or (8b).

III.   THE APPROXIMATION PROBLEM

An ideal lowpass filter has the $|Z_T(j\omega)|^2$ or $|Y_T(j\omega)|^2$ shown in Figure 3. This characteristic is approximated by the function:

$$|Z_T(j\omega)|^2 = 1/1 + F^2(\omega) \text{ or } |Y_T(j\omega)| = 1/1 + F^2(\omega) \tag{13}$$

where $F^2(\omega)$ is a polynomial in ω^2 with real coefficients which is small for $\omega < \omega_c = 1$ and large for $\omega > \omega_c = 1$. (We have normalized the cutoff frequency for convenience.) It can be shown that a realizable minimum reactive network can always be synthesized with a $|Z_T(j\omega)|^2$ of this form. The approximation problem is then to choose $F^2(\omega)$ to approximate the characteristic shown in Figure 3 using some criterion of goodness.

If $F^2(\omega)$ is chosen such that Eq. (13) represents a lowpass filter, then:

$$\left.\begin{array}{c} |Z_T'(j\omega)|^2 \\ \\ |Y_T'(j\omega)|^2 \end{array}\right\} = 1 - 1/1 + F^2(\omega) = F^2(\omega)/1 + F^2(\omega) \tag{14}$$
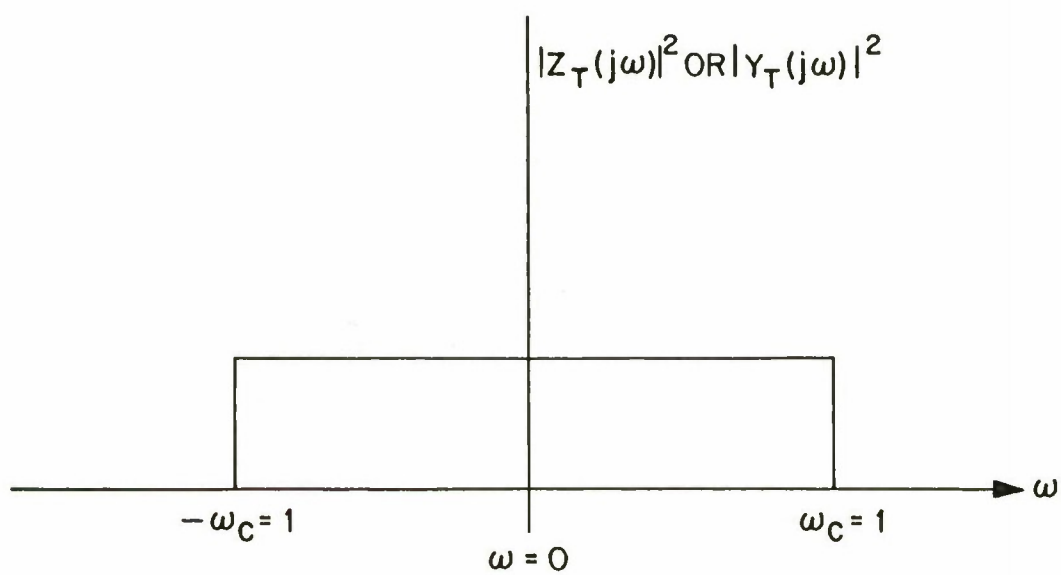
FIG. 3   IDEAL LOW PASS CHARACTERISTIC

represents a highpass filter. This follows from the fact that $|Z_T'(j\omega)|^2$ and $|Z_T(j\omega)|^2$ sum to 1 and, hence, one passes those frequencies which the other does not. We might note that these two networks are complementary and, hence, for the case m = 2, the problem is solved.

Let us now solve the more general case. Let us choose $Z_{Tm}(s)$, the transfer impedance of our m^{th} network, which is a highpass filter such that:

$$|Z_{Tm}(j\omega)|^2 = F^2(\omega/\omega_m)\Big/1 + F^2(\omega/\omega_m) \tag{15}$$

where $F^2(1) = 1$

We have set $F^2(1)$ equal to one so that in Eq. (15) the half power point occurs when $\omega = \omega_m$. Since we are dealing with band separation filters, it is logical to define the passband of each network as the frequency range over which more than half the power is delivered to its load.

Using Eqs. (8a) and (15):

$$\sum_{i=1}^{m-1} |Z_{Ti}(j\omega)|^2 = 1 - \frac{F^2(\omega/\omega_m)}{1 + F^2(\omega/\omega_m)}$$

$$= 1\Big/1 + F^2(\omega/\omega_m) \tag{16}$$

Let us choose $Z_{Tm-1}(s)$ such that

$$1\Big/1 + F^2(\omega/\omega_m) - |Z_{Tm-1}(j\omega)|^2 \tag{17}$$

$$= 1\Big/1 + F^2(\omega/\omega_{m-1})$$

then:

$$|Z_{Tm-1}(j\omega)|^2 = \frac{1}{1+F^2(\frac{\omega}{\omega_m})} - \frac{1}{1+F^2(\frac{\omega}{\omega_{m-1}})} \tag{18}$$

Using Eq. (16),

$$|Z_{Tm-1}(j\omega)|^2 = 1 - \frac{F^2(\omega/\omega_m)}{1+F^2(\omega/\omega_{m-1})} - \frac{1}{1+F^2(\omega/\omega_{m-1})} \tag{19}$$

Z_{Tm-2}, therefore, passes all that is not passed by a highpass filter with cutoff ω_m and a lowpass filter with cutoff ω_{m-1}. $Z_{Tm-1}(j\omega)$ clearly is a bandpass filter with cutoffs ω_{m-1} and ω_m.

Eq. (16) can now be rewritten with the aid of Eq. (17):

$$\sum_{i=1}^{m-2} |Z_{Ti}(j\omega)|^2 = \frac{1}{1+F^2(\omega/\omega_m)} - |Z_{Tm-1}(j\omega)|^2 = \frac{1}{1+F^2(\omega/\omega_{m-1})} \tag{20}$$

This, however, is the same form as Eq. (16). If we let:

$$|Z_{Tm-2}(j\omega)|^2 = \frac{1}{1+F^2(\omega/\omega_{m-1})} - \frac{1}{1+F^2(\omega/\omega_{m-2})} \tag{21}$$

then substitution into Eq. (20) yields:

$$\sum_{i=1}^{m-3} |Z_{Ti}(j\omega)|^2 = 1\Big/1 + F^2(\omega/\omega_{m-2}) \tag{22}$$

It is clear that we can continue this process and, in general,

$$|Z_{Tj}(j\omega)|^2 = 1\Big/1 + F^2(\omega/\omega_{i+1}) - 1\Big/1 + F^2(\omega/\omega_i) \tag{23}$$

and

$$\sum_{i=1}^{m} |Z_{Ti}(j\omega)|^2 = 1 \tag{24}$$

We have thus found a procedure for generating m complementary impedances; the first a lowpass filter, the mth a highpass filter and the rest bandpass filters, all with arbitrary cutoffs. We must now determine for what class of functions $F^2(\omega)$ the $Z_i(s)$ are realizable.

The procedure carried out on the admittance basis yields the result:

$$|Y_{Ti}(j\omega)|^2 = 1\Big/1 + F^2(\omega/\omega_{i+1}) - 1\Big/1 + F^2(\omega/\omega_i) \tag{25}$$

IV. REALIZABILITY CRITERION

We shall restrict out discussion to $|Z_T(j\omega)|^2$ from here on though it applies equally well to an admittance formulation. A given $|Z_T(j\omega)|^2$ corresponds to a realizable network if (a) its poles and zeros are symmetrical about the jω axis and occur in complex conjugate pairs; (b) $|Z_T(j\omega)|^2 \geqq 0$ for all ω.

Condition (a) is guaranteed by restricting $F^2(\omega)$ to the sum of even powers of ω with real coefficients. Condition (b) and Eq. (23) require that:

$$1/1 + F^2(\omega/\omega_{i+1}) \geqq 1/1 + F^2(\omega/\omega_i) \tag{26}$$

or

$$F^2(\omega/\omega_{i+1}) \leqq F^2(\omega/\omega_i) \tag{27}$$

Since by definition $\omega_{i+1} > \omega_i$, this condition is satisfied for any $F^2(\omega)$, which is a monotonic increasing function of ω. This condition is not a necessary one since the condition given in Eq. (27) need not be satisfied everywhere but only at $m+1$ points. If the solution, however, is to be applicable to any arbitrary set of cutoff frequencies, this condition is necessary.

We have thus found a procedure for choosing the m networks whose input impedances sum to one and whose transfer impedances have the desired bandpass characteristics. We have shown that if $F^2(\omega)$ is a monotone, increasing function of ω, the networks are realizable. We must now evaluate the performance for the various $F^2(\omega)$ which when used in Eq. (16) approximate the ideal lowpass characteristic and which satisfy this condition.

V. THE BUTTERWORTH CHARACTERISTIC

The Butterworth lowpass characteristic of nth order is given by:

$$|Z_T(j\omega)|^2 = 1/1 + \omega^{2n} \tag{28}$$

Hence, by our previous notation,

$$F^2(\omega) = \omega^{2n} \tag{29}$$

ω^{2n} is clearly an increasing function of ω and hence from Eqs. (23) or (25):

$$\left.\begin{array}{c} |Z_{Ti}(j\omega)|^2 \\ |Y_{Ti}(j\omega)|^2 \end{array}\right\} = 1 \Big/ 1+(\omega/\omega_{i+1})^{2n} - 1\Big/1+(\omega/\omega_i)^{2n} \tag{30}$$

Let us calculate the minimum insertion loss in the passband using Eq. (30). The minimum insertion loss occurs when:

$$\frac{d}{d\omega}\left[\frac{1}{1+(\omega/\omega_{i+1})^{2n}} - \frac{1}{1+(\omega/\omega_i)^{2n}}\right] = 0 \tag{31}$$

differentiating

$$\frac{2n\omega^{2n-1}}{\omega_{i+1}^{2n}\left[1+(\omega/\omega_{i+1})^{2n}\right]^2} = \frac{2n\omega^{2n-1}}{\omega_i^{2n}\left[1+(\omega/\omega_i)^{2n}\right]^2} \tag{32}$$

or

$$\left[\omega_{i+1}^n + \omega^{2n}/\omega_{i+1}^n\right]^2 = \left[\omega_i^n + \omega^{2n}/\omega_i^n\right]^2$$

Solving for ω yields:

$$\omega^2 = \omega_i\omega_{i+1} \tag{33}$$

This value of ω corresponds to the minimum insertion loss or maximum value of $|Z_{Ti}(j\omega)|^2$ given by:

$$|Z_{Ti}(j\omega)|^2_{max} = 1\Big/1+(\omega_i/\omega_{i+1})^n - 1\Big/1+(\omega_{i+1}/\omega_i)^n \tag{34}$$

and

$$\lim_{(\omega_i/\omega_{i+1})^n \to 0} |Z_{Ti}(j\omega)|^2_{max} = 1 \tag{35}$$

At the cutoff frequencies $\omega = \omega_i$ and $\omega = \omega_{i+1}$

$$|Z_{Ti}(j\omega_i)|^2 = \frac{1}{1 + (\omega_i/\omega_{i+1})^{2n}} - 1/2 \tag{36}$$

$$|Z_{Ti}(j\omega_{i+1})|^2 = 1/2 - \frac{1}{1 + (\omega_{i+1}/\omega_i)^{2n}} \tag{37}$$

Subtracting Eq. (37) from Eq. (36) yields:

$$|Z_{Ti}(j\omega_i)|^2 - |Z_{Ti}(j\omega_{i+1})|^2 = 0 \tag{38}$$

Hence:

$$|Z_{Ti}(j\omega_i)|^2 = |Z_{Ti}(j\omega_{i+1})|^2 \tag{39}$$

and it follows from Eqs. (36) and (37) that:

$$\lim_{(\omega_i/\omega_{i+1})^n \to 0} |Z_{Ti}(j\omega)|^2 = \lim_{(\omega_i/\omega_{i+1})^n \to 0} |Z_{Ti}(j\omega_{i+1})|^2 = 1/2 \tag{40}$$

We, therefore, see that in the limit a minimum insertion loss of zero db and a 3 db insertion loss at the cutoffs can be achieved. In Figure 4 Eqs. (34) and (36) are plotted. From these plots:

$$\left| Z_{Ti}(j\omega) \right|^2_{max} \text{ and } \left| Z_{Ti}(j\omega_i) \right|^2 = \left| Z_{Ti}(j\omega_{i+1}) \right|^2$$

can be determined. This gives some idea of the performance that can be achieved for values of $(\omega_{i+1}/\omega_i)^n$. If we require a minimum insertion loss of less than 2 db, then:

$$\left| Z_{Ti}(j\omega) \right|^2_{max} > 0.632 \tag{41}$$

From Figure 4

$$(K)^n = (\omega_{i+1}/\omega_i)^n > 4.4 \tag{42}$$

In Table I is shown values of n required for various values of K to satisfy Eqs. (41) and (42).

K	1.1	1.3	1.5	2.0
n	16	6	4	3

Table I

Values of n and K for Minimum Insertion Loss of less than 2 db for Butterworth-Type Band Separation Filter.
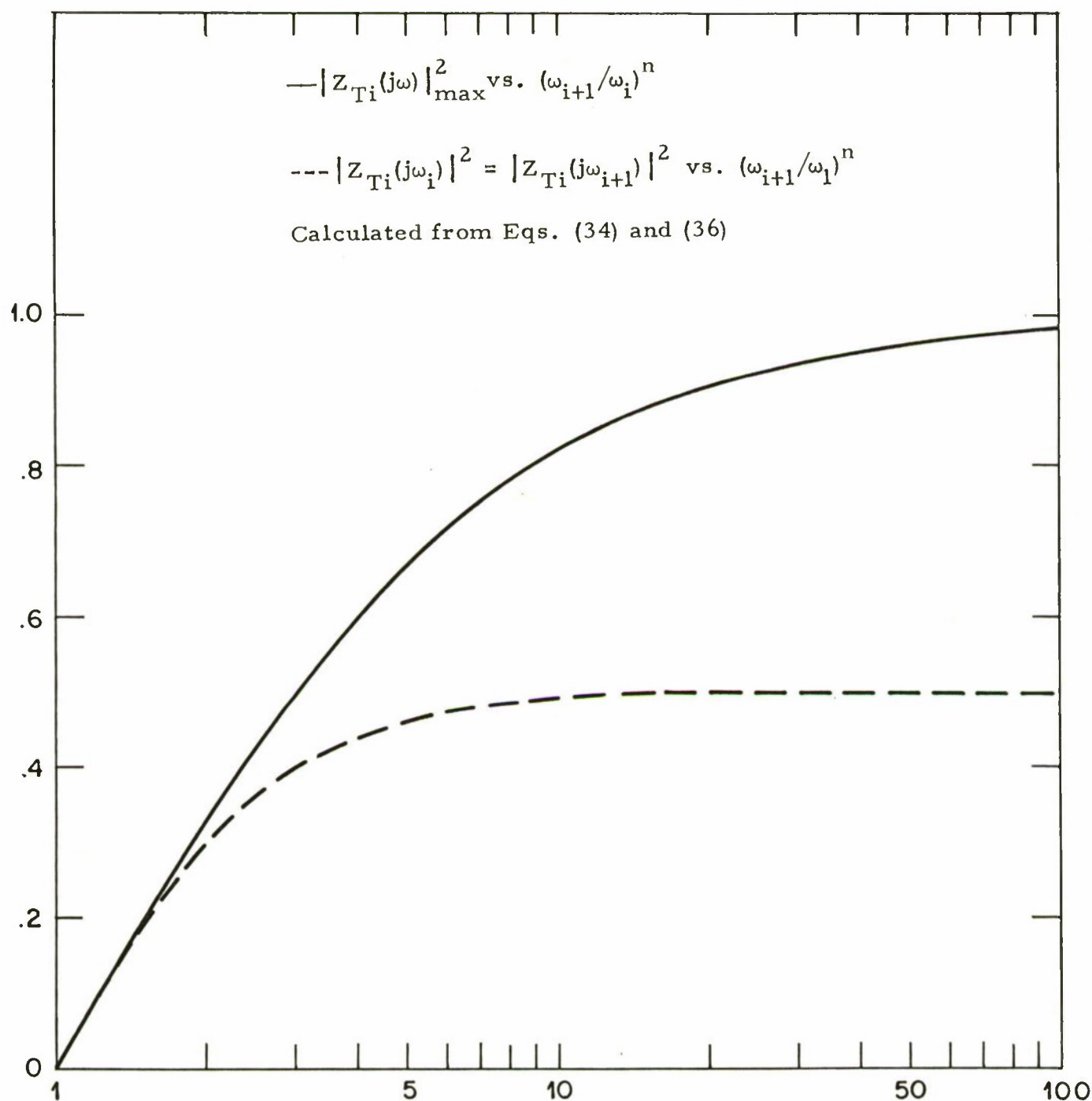
FIG. 4 MINIMUM INSERTION LOSS AND INSERTION LOSS AT CUTOFF FOR BUTTERWORTH TYPE BAND SEPARATION FILTER

In Figures 5, 6, 7 and 8, Eq. (30) is plotted for n = 4, 8 and 16 and K = 1.1, 1.3, 1.5 and 2.0 respectively. $|Z_{Ti}(j\omega)|^2$ is plotted in these curves vs. ω_i + RW where R takes on values between -1 and +2 and W equal to $\omega_{i+1} - \omega_i$ is the nominal bandwidth. A second frequency scale is given in terms of fractions of ω_i.

As can be seen from Figure 5 for K = 1.1, n = 16 yields the only usable characteristic as predicted in Table I. It should be noted that the actual cutoffs; i.e., 3 db points, occur at 1.006ω_i and 1.098ω_i and K = 1.09. The deviation is thus small, but can be compensated for if desired by choosing ω_i and ω_{i+1} slightly different from the desired cutoff.

Figures 6, 7 and 8 indicate that for K = 1.3, n = 8 and 16 yield usable characteristics while for K > 1.5, n = 4 is also usable. It is clear that the larger the value of K^n, the better the characteristic.

The asymptotic behavior of $|Z_{Ti}(j\omega)|^2$ can be seen from Eq. (30) as $\omega \to \infty$

$$\lim_{\omega \to \infty} |Z_{Ti}(j\omega)|^2 = \frac{(\omega_{i+1})^{2n}}{\omega^{2n}} - \frac{(\omega_i)^{2n}}{\omega^{2n}} = \frac{(\omega_{i+1})^{2n} - (\omega_i)^{2n}}{\omega^{2n}} \quad (43)$$

Expressing this as a loss in db,

$$I_{db} = 20 \log C\omega^n = 20 (\log C + \log \omega^n) \quad (44)$$

where

$$C = \left[\frac{1}{(\omega_{i+1})^{2n} - (\omega_i)^{2n}}\right]^{1/2}$$

let

FIG. 5. NORMALIZED TRANSFER IMPEDANCE BUTTERWORTH TYPE BAND
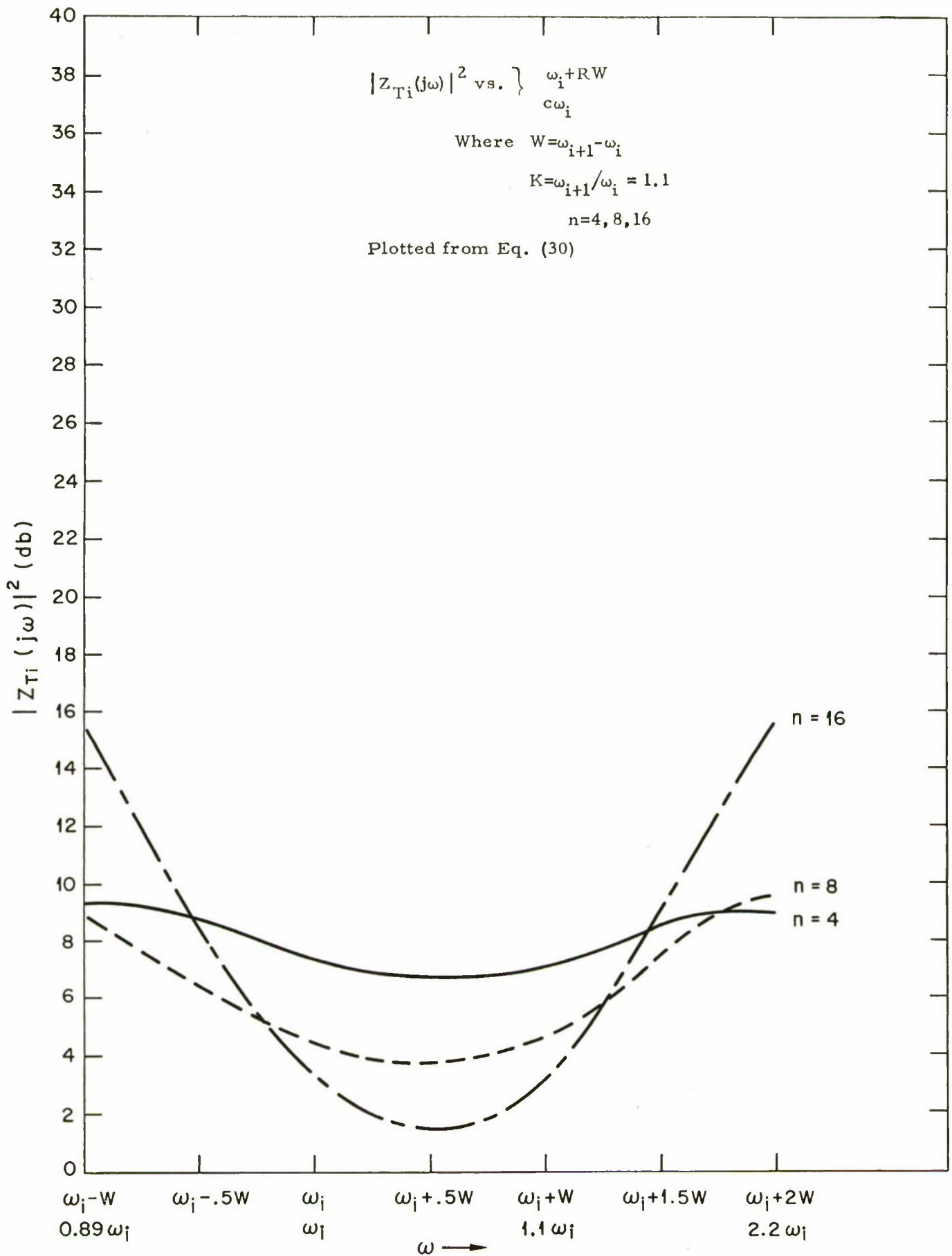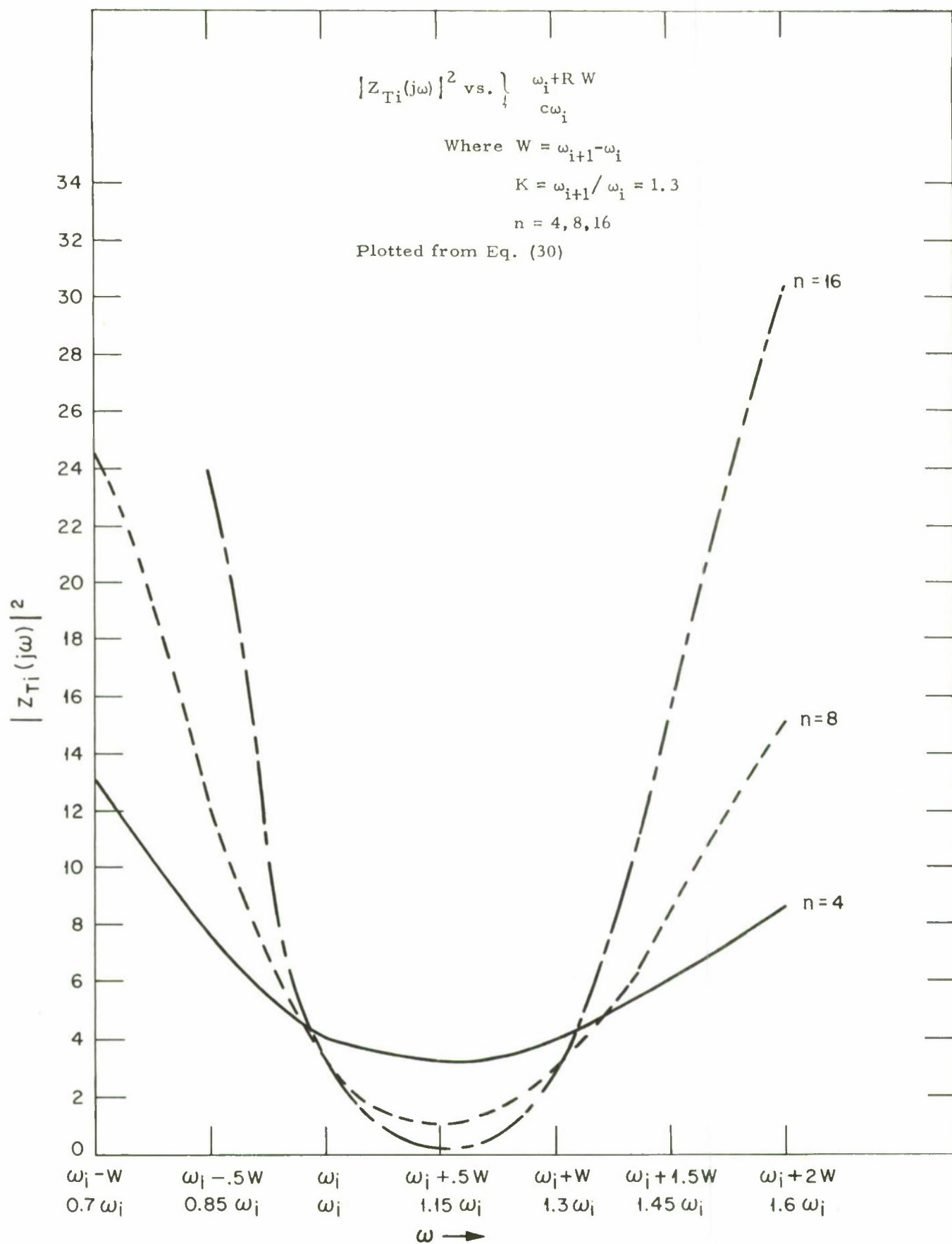SEPARATION FILTERS K = 1.1

FIG. 6   NORMALIZED TRANSFER IMPEDANCE BUTTERWORTH TYPE
BAND SEPARATION FILTERS K=1.3.

B-12849-16
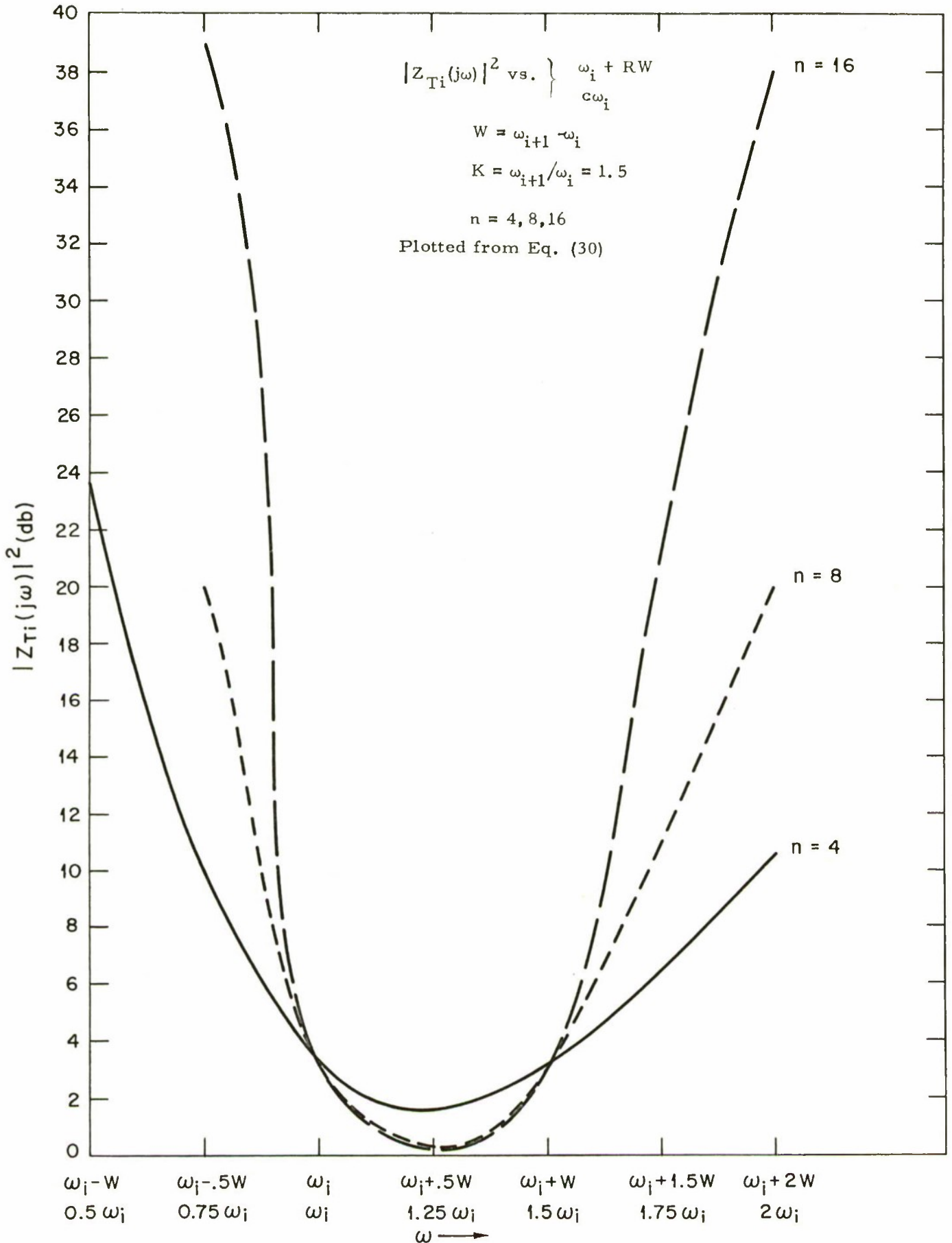
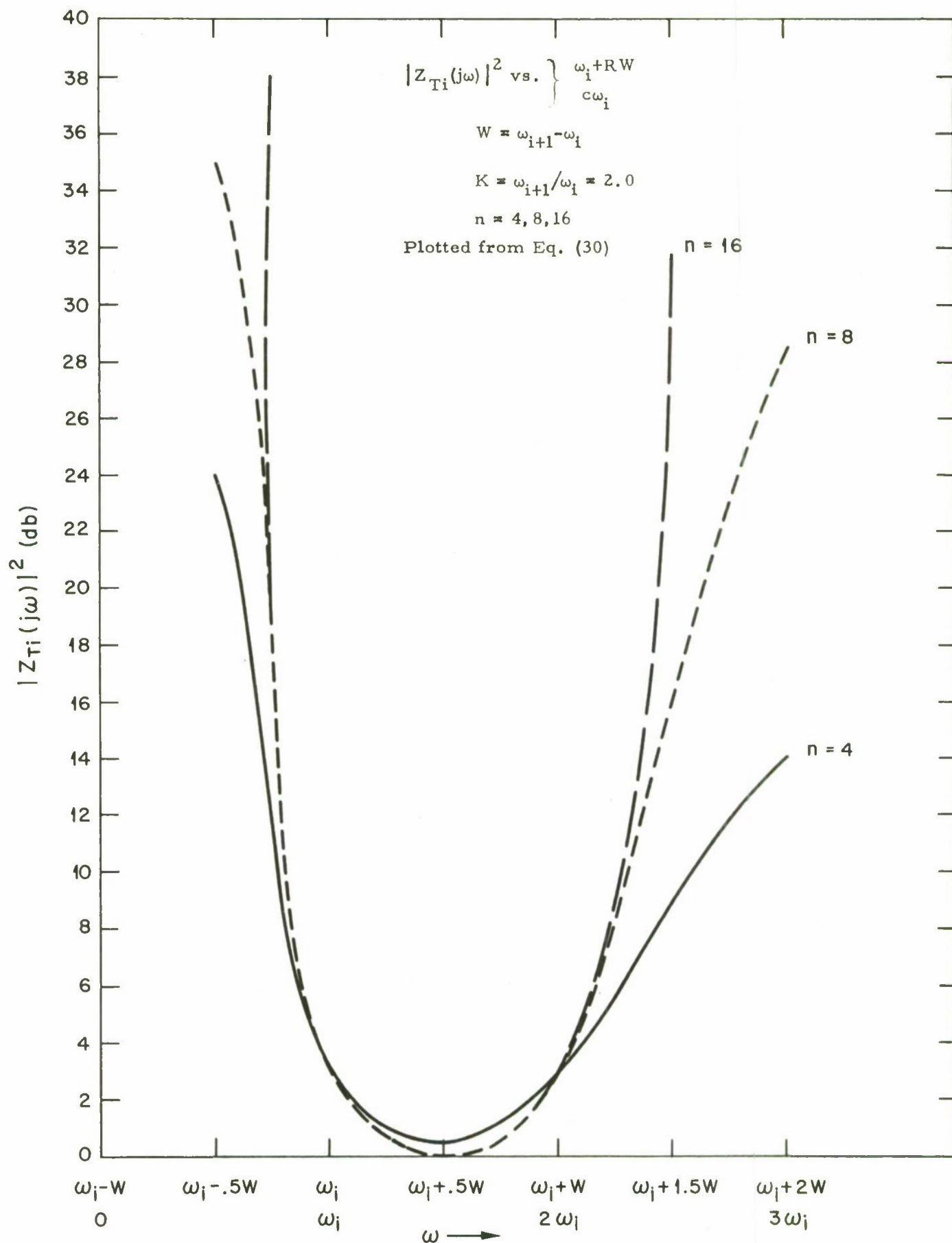FIG. 7. NORMALIZED TRANSFER IMPEDANCE BUTTERWORTH TYPE BAND SEPARATION FILTERS K = 1.5

FIG. 8. NORMALIZED TRANSFER IMPEDANCE BUTTERWORTH TYPE BAND SEPARATION FILTERS K = 2.0

B-12849-12

$$\omega = 2^{\mu} \qquad\qquad K = 20 \log C$$

$$I_{db} = K + 20 \log^{\mu n} \approx K + 20 \, \mu n \log 2 \approx K + 6 \, n\mu \qquad (45)$$

Hence this function has a slope of 6n db per active and an intercept of K at $\mu = 0$.

VI.   THE PAPOULIS CHARACTERISTIC

The Butterworth filter has the property of being maximally flat at the center of its passband; however, its rate of cutoff is relatively slow.   To reduce the number of elements required for a given value of K, it is desirable to find that $F^2(\omega)$ which is monotonic and cuts off fastest.   Papoulis[1] has derived a class of filters called L filters which have the following property:

$$F^2(\omega) = L_n(\omega^2) \qquad (46)$$

where

(a) $L_n(0) = 0$

(b) $L_n(1) = 1$ $\qquad\qquad (47)$

(c) $\left. \dfrac{dL_n(\omega^2)}{d\omega} \right|_{\omega = 1} = M$

where M is the largest value obtainable for any polynomial in even powers of ω of order 2n satisfying (a) and (b).

Table II lists $L_n(\omega^2)$ for n = 2, 3, 4, 5, 6, 7 and 8.

n	$n(\omega^2)$
2	ω^4
3	$3\omega^6 - 3\omega^4 + \omega^2$
4	$6\omega^8 - 8\omega^6 + 3\omega^4$
5	$20\omega^{10} - 40\omega^8 + 28\omega^6 - 8\omega^4 + \omega^2$
6	$50\omega^{12} - 120\omega^{10} + 105\omega^8 - 40\omega^6 + 6\omega^4$
7	$175\omega^{14} - 525\omega^{12} + 615\omega^{10} - 355\omega^8 + 105\omega^6 - 15\omega^4 + \omega^2$
8	$490\omega^{16} - 1680\omega^{14} + 2310\omega^{12} - 1624\omega^{10} + 615\omega^8 - 120\omega^6 + 10\omega^4$

Table II

$L_n(\omega^2)$ for n = 2, 3, 4, 5, 6, 7 and 8. [1,2]

Eq. (23) can be rewritten:

$$|Z_{Ti}(j\omega)|^2 = \frac{1}{1 + L_n\left(\frac{\omega}{\omega_{i+1}}\right)^2} - \frac{1}{1 + L_n\left(\frac{\omega}{\omega_i}\right)^2} \tag{48}$$

The passband characteristic given in Eq. (48) can be evaluated by first evaluating $L_n(\omega^2)$. For a given value of K, one must evaluate $L_n(\omega/K\omega_i)^2$ and $L_n(\omega/\omega_i)^2$ for different values of ω. These values are then substituted into Eq. (48) to determine $|Z_{Ti}(j\omega)|^2$. Figure 9 is a plot of $|Z_{Ti}(j\omega)|^2$ for n = 4 and K = 1.3, 1.5 and 2.0, while in Figure 10, n = 8 and K = 1.1, 1.3, 1.5 and 2.0.
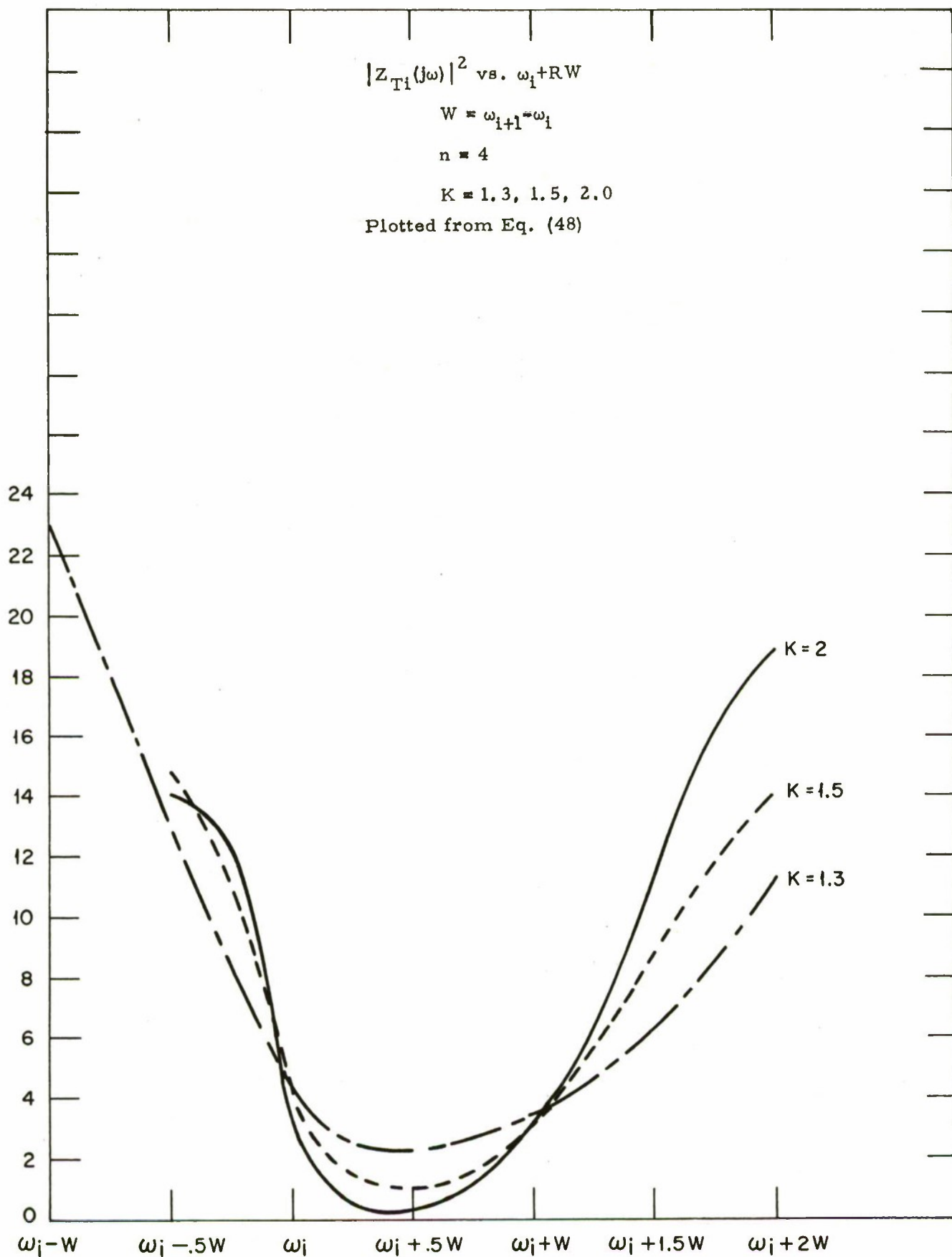
FIG. 9   NOMALIZED TRANSFER IMPEDANCE L TYPE BAND
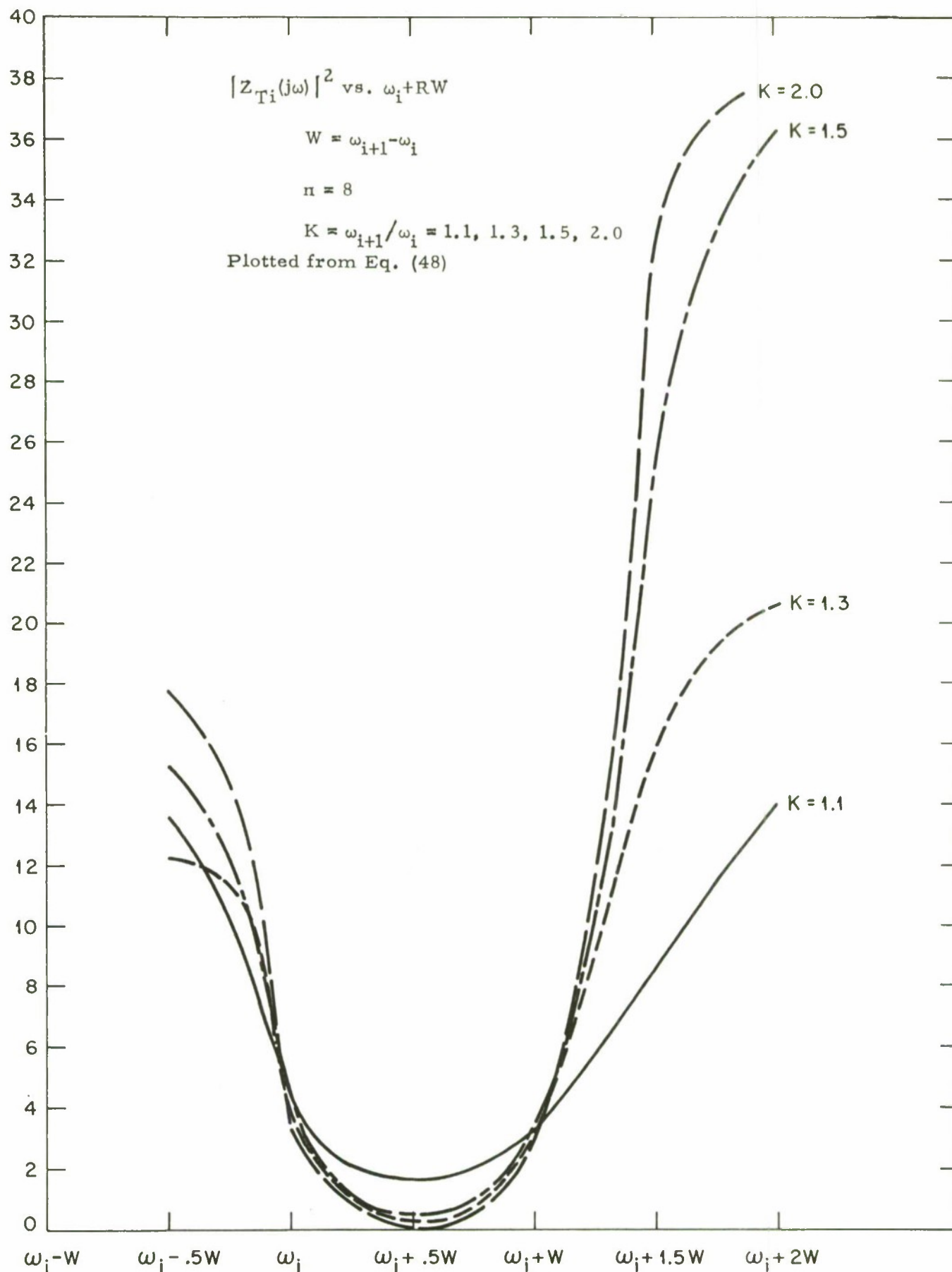SEPARATION FILTER n = 4.

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FIG. 10 NORMALIZED TRANSFER IMPEDANCE L-TYPE BAND
SEPARATION FILTER   n = 8

B-128.49-18

From Figure 10 we note that for K = 1.1 n = 8 the characteristic is usable by the same criterion by which n = 16 was necessary for the Butterworth. Further comparison is shown in Figure 11 for K = 1.3. Here we note that the fourth order L filter is comparable (though not quite as good) to the eighth order Butterworth and the sixteenth order L is comparable to the eighth order Butterworth. We note that this statement is true in the passband and upper stopband. In the lower stopband, the characteristic deteriorates somewhat and for very low frequencies it is not as good as the Butterworth. The reason for this is clear upon examining Eq. (48). At high frequencies, both terms on the right approach zero at the maximum rate; hence, their difference approaches zero at least as fast as each term. As we approach zero frequency, each term approaches 1 and only their difference approaches zero. How fast it cuts off near zero is determined by how fast each term approaches 1. The Butterworth, we recall, is maximally flat and, hence, approaches one quickly. The L filters, on the other hand, are concerned with the slope at cutoff and, hence, approach one at zero frequency slowly. We thus see that for uniform selectivity for both high and low frequencies, the Butterworth is desirable. If one is interested in economizing on the number of elements and wants a sharp cutoff to 10 or 12 db points, the L filters should be used. As seen from Figures 9, 10 and 11, the L filters give faster cutoffs and lower passband insertion loss than Butterworth but do not have as high an attenuation at the low frequencies.

VII.   SYNTHESIS PROCEDURE FOR BUTTERWORTH NETWORKS

Eq. (30) can be rewritten:

$$|Z_{Ti}(j\omega)|^2 = \frac{A\omega^{2n}}{\left[1 + (\frac{\omega}{\omega_{i+1}})^{2n}\right]\left[1 + (\frac{\omega}{\omega_i})^{2n}\right]} \qquad (49)$$
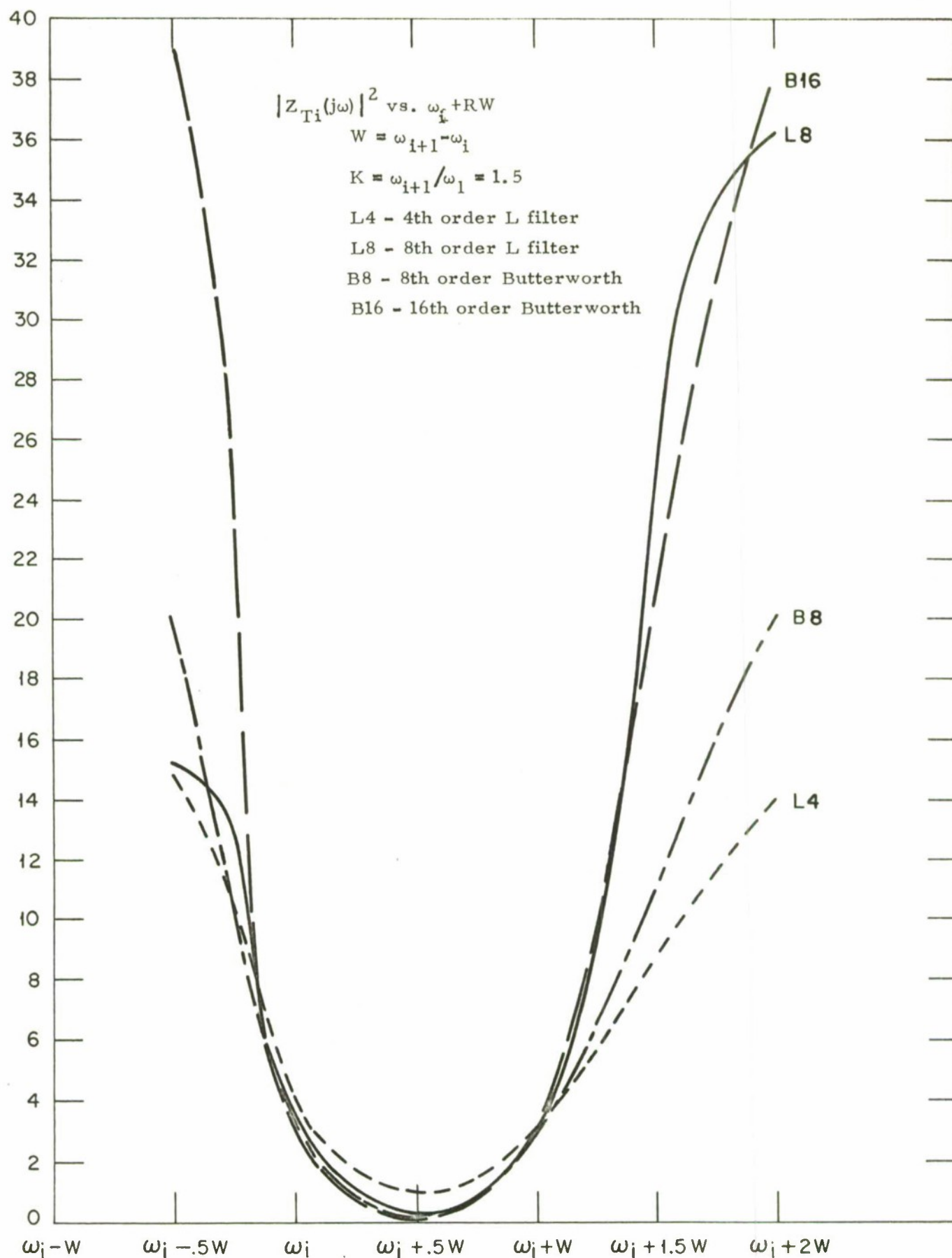
FIG. 11   COMPARISON OF BUTTERWORTH AND L TYPE BAND SEPARATION
FILTER FOR K = 1.3

B-12849-15

where

$$A = \left[\frac{1}{\omega_i^{2n}} - \frac{1}{\omega_{i+1}^{2n}} \right]$$

It is well known that a transfer impedance of this form can be synthesized as a ladder network.[3] Darlington has shown for a lossless network terminated in a one ohm resistance whose input impedance $Z(s)$ is given by:

$$Z(s) = (m_1 + n_1)/(m_2 + n_2) \tag{50}$$

where m_1 and m_2 are even polynomials and n_1 and n_2 are odd that the open circuit impedances of the lossless network are given by:

Case A

$$z_{11} = m_1/n_2$$

$$z_{22} = m_2/n_2$$

$$z_{12} = \sqrt{m_1 m_2 - n_1 n_2}/n_2$$

Case B

$$z_{11} = n_1/m_2$$

$$z_{22} = n_2/m_2 \tag{51}$$

$$z_{12} = \sqrt{n_1 n_2 - m_1 m_2}/m_2$$

where the correct case is determined by the condition that z_{12} must be the quotient of an even polynomial over an odd or odd over even. If $\sqrt{m_1 m_2 - n_1 n_2}$ is even, case A is used; if odd, case B is used. If $\sqrt{m_1 m_2 - n_1 n_2}$ is not a perfect square, it must be augmented by multiplying numerator and denominator of $Z(s)$ by a suitable polynomial. [3] By evaluating the residues in Eq. (51), it is easily verified that the residue condition is satisfied with the equal sign. For case A:

$$k_{11} = \left. \frac{m_1}{n_2'} \right|_{s=s_a} \qquad k_{22} = \left. \frac{m_2}{n_2'} \right|_{s=s_a} \qquad k_{12} = \left. \frac{\sqrt{m_1 m_2}}{n_2'} \right|_{s=s_a} \tag{52}$$

where s_a is a zero of n_2 and the prime indicates differentiation with respect to s. [4]

Hence:

$$k_{11} k_{22} - k_{12}^2 = 0 \tag{53}$$

To synthesize the minimum reactive network corresponding to $|Z_{Ti}(j\omega)|^2$, determine $m_2 + n_2$ and hence, z_{22}. If a lossless network with this z_{22} has the transmission zeros of $\sqrt{m_1 m_2 - n_1 n_2}$ and satisfies the residue condition with the equal sign, it must be the desired network within an impedance scaling factor. The impedance scaling factor arises from the fact that if $Z(s)$ is multiplied by a constant K^2, $z_{22} = m_2/n_2$ or n_2/m_2 is not affected, but z_{12} is multiplied by K. Hence, this constant must be determined and the impedance levelled to get the correct constant A in Eq. (49).

Since half the transmission zeros of Eq. (49) are at $\omega = 0$ and half at $\omega = \infty$, z_{22} must be developed in a ladder network with half its transmission zeros at $\omega = 0$ and half at $\omega = \infty$. If this is done by complete removal of each pole, the residue condition is satisfied with the equal sign, and upon appropriate scaling, the desired network is achieved.

$m_2 + n_2$ can be found as follows:

$$|Z_{Ti}(j\omega)|^2 = \text{Re}\,[Z(s)]_{s=j\omega} = \left. \frac{m_1 m_2 - n_1 n_2}{(m_2 + n_2)(m_2 - n_2)} \right|_{s=j\omega} \tag{54}$$

Since the poles of $Z(s)$ are all in the left half plane, the product of the LHP poles of the Re $[Z(s)]$ must be equal to $m_2 + n_2$. Hence, by factoring the denominator of Eq. (49) and taking the left half plane zeros, $m_2 + n_2$ is determined.

The determination of $m_2 + n_2$ for $|Z_{Ti}(j\omega)|^2$ of the form given in Eq. (49) can be simplified as follows:

Let s_1^i, $s_2^i, \ldots, s_n^i$ be the zeros of the polynomial $[1 + (-js/\omega_i)^{2n}]$ lying in the LHP and s_1^{i+1}, $s_2^{i+1}, \ldots, s_n^{i+1}$ be the zeros of the polynomial $[1 + (-js/\omega_{i+1})^{2n}]$ lying in the LHP. Then,

$$m_2 + n_2 = \left[\prod_{K-1}^{n} (s - s_K^i) \right] \left[\prod_{\ell=1}^{n} (s - s_e^{i+1}) \right] = (m_2^i + n_2^i)(m_2^{i+1} + n_2^{i+1}) \quad (55)$$

where

$$m_2^i + n_2^i = \prod_{K=1}^{n} (s - s_K^i) \quad (56a)$$

$$m_2^{i+1} + n_2^{i+1} = \prod_{\ell=1}^{n} (s - s_\ell^{i+1}) \quad (56b)$$

Let the zeros of $[1 + (-js)^{2n}]$ be s_1^o, $s_2^o, \ldots, s_n^o$, then

$$m_2^o(s) + n_2^o(s) = \prod_{K=1}^{n} (s - s_K^o) \quad (57)$$

and

$$m_2^i(s) + n_2^i(s) = m_2^o(s/\omega_i) + n_2^o(s/\omega_i) \quad (58a)$$

$$m_2^{i+1}(s) + n_2^i(s) = m_2^o(s/\omega_{i+1}) + n_2^o(s/\omega_{i+1}) \quad (58b)$$

The polynomials $m_2^o + n_2^o$ are just the denominator of the frequency normalized Butterworth function and are well known. The polynomials for $n = 1, 2, \ldots, 8$ are given in Table III.[5] From this table $m_2 + n_2$ is easily determined using Eqs. (58) and (55).

n	
1	$1 + S$
2	$1 + 1.414S + S^2$
3	$1 + 2S + 2S^2 + S^3$
4	$1 + 2.613S + 3.414S^2 + 2.613S^3 + S^4$
5	$1 + 3.236S + 5.236S^2 + 5.236S^3 + 3.236S^4 + S^5$
6	$1 + 3.864S + 7.464S^2 + 9.141S^3 + 7.464S^4 + 3.864S^5 + S^6$
7	$1 + 4.494S + 10.103S^3 + 14.606S^3 + 14.606S^4 + 10.103S^5 + 4.494S^6 + S^7$
8	$1 + 5.126S + 13.138S^2 + 21.848S^3 + 25.691S^4 + 21.848S^5 + 13.138S^6 + 5.126S^7 + S^8$

Table III

Denominator Polynomial Butterworth Network[5]

Example I:

Let us consider as an example a three-way crossover network for a high fiedlity system. The first filter will pass zero to 4000 cycles; the second, 4000 to 8000 cycles and the third, all frequencies greater than 8000. The desired impedance level is 8 ohms. Let us normalize to 1 ohm and let 8000 cycles correspond to $\omega = 1$. Using fourth order Butterworth functions:

$$|z_{T1}(j\omega)|^2 = 1/1 + (2\omega)^8 \qquad (59a)$$

$$|z_{T2}(j\omega)|^2 = 1\big/1+\omega^8 - 1\big/1+(2\omega)^8 \qquad (59b)$$

$$|z_{T3}(j\omega)|^2 = \omega^8/1+\omega^8 \qquad (59c)$$

Let us synthesize the highpass network first. From Table III, $n = 4$

$$m_2^3 + n_2^3 = 1 + 2.613s + 3.414s^2 + 2.613s^3 + s^4 \qquad (60)$$

Since $m_1m_2 - n_1n_2 = s^4$ is even

$$z_{22} = \frac{1 + 3.414s^2 + s^4}{2.613s + 2.613s^3} \qquad (61)$$

z_{22} must be developed in a ladder network with all transmission zeros at $\omega = 0$.

$$
\require{enclose}
\begin{array}{r}
.383/s \\[2pt]
2.613s+2.613s^3\,\enclose{longdiv}{1+3.414s^2+s^4} \\
\underline{1\qquad s^2} \\
2.414s^2+s^4\,\enclose{longdiv}{2.613s+2.613s^3}\quad 1.082/s \\
\underline{2.613s+1.082s^3}\quad 1.57/s \\
1.531s^3\,\enclose{longdiv}{2.414s^2+s^4} \\
\underline{2.414s^2}\quad 1.531/s \\
s^4\,\enclose{longdiv}{1.531s^3}
\end{array}
$$

The resulting lossless network is shown in Figure 12.
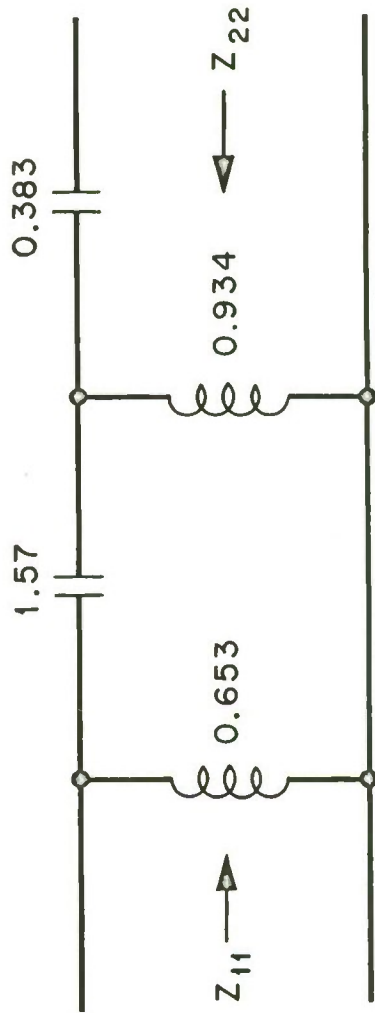
FIG.12 HIGH PASS NETWORK OF EXAMPLE I

Jesulaitis Fig. GRAYZEL A-12849-9

To evaluate the constant multiplier to determine if impedance scaling is necessary, $z_{12}(s)$ is evaluated at $s = \infty$. Since by Eq. (51),

$$z_{12}(s) = \frac{m_1 m_2 - n_1 n_2}{n_2} = \frac{s^4}{2.613(s+s^3)} \tag{62}$$

$$z_{12}(\infty) \rightarrow \frac{s}{2.613} = 0.384s \tag{63}$$

At infinite frequency, the circuit of Figure 12 reduces to that of Figure 13.

It is clear from Figure 13b that $z_{12}(\infty) = 0.384$ and, hence, no impedance levelling is necessary.

Let us next synthesize the lowpass network. From Table III, using Eq. (58),

$$m_2^1 + n_2^1 = 1 + (2.613)(2s) + (3.414)(2s)^2 + (2.613)(2s)^3 + (2s)^4 \tag{64}$$

$$= 1 + (5.226)s + (13.656)s^2 + (20.90)s^3 + 16s^4$$

Since $\sqrt{m_1 m_2 - n_1 n_2} = 1$ is even,

$$z_{22} = \frac{1 + 13.656s^2 + 16s^4}{5.226s + 20.904s^3} \tag{65}$$

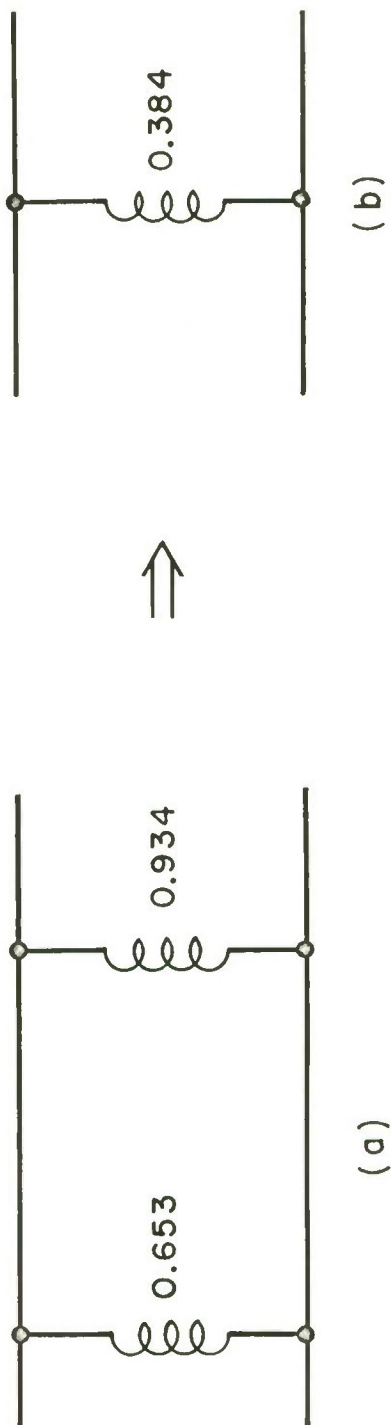z_{22} must be developed in a ladder network with all of its transmission zeros at infinity.

0.384

(b)

0.934

(a)

0.653

FIG. 13 ASSYMTOTIC BEHAVIOR OF NETWORK FIG. 12 AS $\omega \rightarrow \infty$

$$\begin{array}{r} 0.766s \\ 20.904s^2+5.226s \,\big|\, \overline{16s^4 + 13.656s^2+1} \\ 16s^4 + 4.030s^2 \end{array}$$

$$19.626s^2+1 \,\big|\, \overline{20.904s^3+5.226s} \qquad 1.063s$$
$$20.904s^3+1.063s \qquad 4.72s$$
$$4.163s \,\big|\, \overline{19.626s^2+1}$$
$$19.626s^2 \qquad 4.163s$$
$$1 \,\big|\, \overline{4.163s}$$

The resulting lossless network is shown in Figure 14.

To evaluate the constant multiplier of $z_{12}(s)$ to determine if impedance scaling is required $z_{12}(s)$ is evaluated at $s = 0$.

Since

$$z_{12}(s) = 1/5.266s + 20.904s^3 \qquad (66)$$

$$z_{12}(0) = 1/5.266s \qquad (67)$$

At zero frequency the network of Figure 14 reduces to that shown in Figure 15.

It is clear from Figure 15b that $z_{12}(0) = 1/5.226s$ and, hence, no scaling is necessary.

Now let us synthesize the bandpass network. From Eq. (58)

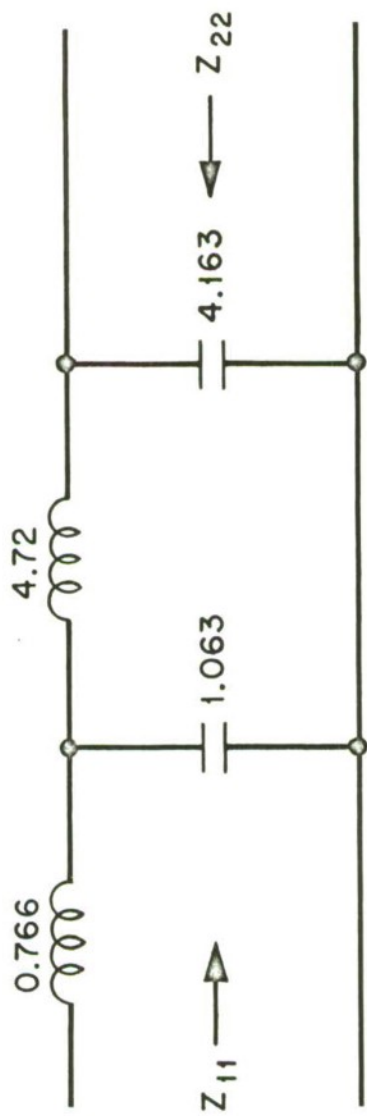$$m_2^2 + n_2^2 = (m_2^1 + n_2^1)(m_2^3 + n_2^3) \qquad (68)$$

FIG.14 LOW PASS NETWORK OF EXAMPLE I
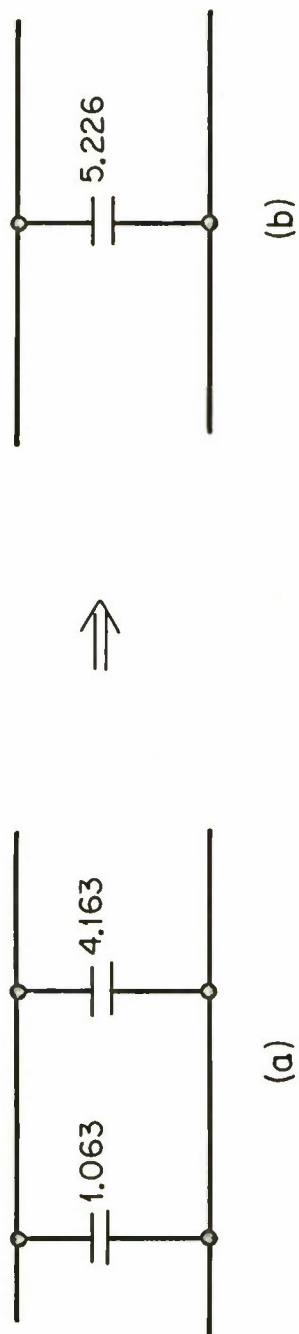
Jesulaitis Fig. GRAYZEL A-128849-7

FIG 15 ASSYMTOTIC BEHAVIOR OF NETWORK FIG. 14 AS $\omega \rightarrow 0$

(a)

1.063

4.163

(b)

5.226

Using Eqs. (60) and (64)

$$m_2^2 + n_2^2 = 1 + 7.8s + 30.7s^2 + 77.0s^3 + 131.9s^4 + 154.1s^5 + 123.1s^6 + 62.7s^7 + 16s^8$$

Since

$$|Z_{T2}(j\omega)|^2 = 1/1+\omega^8 - 1/1+(2\omega)^8 = \frac{(2^8-1)\omega^8}{[1+\omega^8][1+(2\omega)^8]} \qquad (69)$$

$$\sqrt{m_1 m_2 - n_1 n_2} = \sqrt{2^8-1} \quad s^4 \text{ is even} \qquad (70)$$

and

$$z_{22} = \frac{1 + 30.7s + 131.9s^4 + 123.0s^6 + 16.0s^8}{7.8s + 77.0s^3 + 154.1s^5 + 62.7s^7} \qquad (71)$$

z_{22} must be developed in a ladder network with four transmission zeros at infinity and four at zero frequency.

$$7.85 + 77.0s^3 + 154.1s^5 + 62.7s^7 \overline{\smash{\big)}\ \substack{1/7.8s \\ 1 + 30.7s^2 + 131.9s^4 + 123.0s^6 + 16.0s^8}}$$

$$1 + 9.9s^2 + 19.8s^4 + 8.1s^6$$

$$\overline{20.8s^2 + 112.1s^4 + 114.9s^6 + 16.0s^8}$$

$$20.8s^2 + 112.1s^4 + 114.9s^6 + 16s^8 \overline{\smash{\big)}\ \substack{1/2.67s \\ 7.8s + 77.0s^3 + 154.1s^5 + 62.7^7}}$$

$$7.8s + 42.1s^3 + 43.0s^5 + 6.0s^7$$

$$\overline{34.9s^3 + 111.1s^5 + 56.7s^7}$$

$$34.9s^3 + 111.1s^5 + 56.7s^7 \overline{\smash{\big)}\ \substack{1/1.68s \\ 20.8s^2 + 112.1s^4 + 114.9s^6 + 16s^8}}$$

$$20.8s^2 + 66.3s^4 + 33.8s^6$$

$$\overline{45.8s^4 + 81.1s^6 + 16s^8}$$

$$45.8s^4 + 81.1s^6 + 16s^8 \overline{\smash{\big)}\ \substack{1/1.31s \\ 34.9s^3 + 111.1s^5 + 56.7s^7}}$$

$$34.9s^3 + 62.0s^5 + 12.2s^7$$

$$\overline{49.1s^5 + 44.5s^7}$$

$$44.5s^7 + 49.1s^5 \overline{\smash{\big)}\ \substack{.359s \\ 16s^8 + 81.1s^6 + 45.8s^4}}$$

$$16s^8 + 17.7s^6$$

$$\overline{63.4s^6 + 45.8s^4} \overline{\smash{\big)}\ \substack{.718s \\ 44.5s^7 + 49.1s^5}}$$

$$44.5s^7 + 32.9s^5$$

$$\overline{16.2s^5}$$

$$16.2s^5 \overline{\smash{\big)}\ \substack{3.91s \\ 63.4s^6 + 45.8s^4}}$$

$$63.4s^6$$

$$\overline{45.8s^4} \overline{\smash{\big)}\ \substack{.355s \\ 16.25s^5}}$$

The resulting lossless network is shown in Figure 16.

To evaluate the constant multiplier of $z_{12}(s)$ to determine if impedance scaling is necessary, $z_{12}(s)$ is evaluated at s = 0. Using Eqs. (68) and (70)

$$z_{12}(s) = \frac{\sqrt{m_1 m_2 - n_1 n_2}}{n_2} = \frac{\sqrt{2^8 - 1} \ s^4}{n_2} \tag{72}$$

and

$$z_{12}(0) = \frac{\sqrt{2^8 - 1} \ s^4}{7.8s} = 2.05 \, s^3 \tag{73}$$

At zero frequency, Figure 16 reduces to that shown in Figure 17.

$z_{12}(0)$ can now be determined as follows: Assume an output voltage E_{cd} of 1 volt, then,

(a) $E_{bd} = 1$

(b) $I_{bd} = 1/2.67s$

(c) $E_{ab} = 1/2.67s \times 1/1.68s \approx E_{ad}$ (74)

(d) $I_{ad} \approx 1/2.67s \times 1/1.68s \times 1/1.31s = 1/5.86s$

(e) $z_{12}(0) \approx E_{bd}/I_{ad} = 5.86s$

The impedance level is thus seen to be too large and must be scaled by K where

$$K = 2.05/5.86 = 0.35 \tag{75}$$

The desired network is shown in Figure 18.

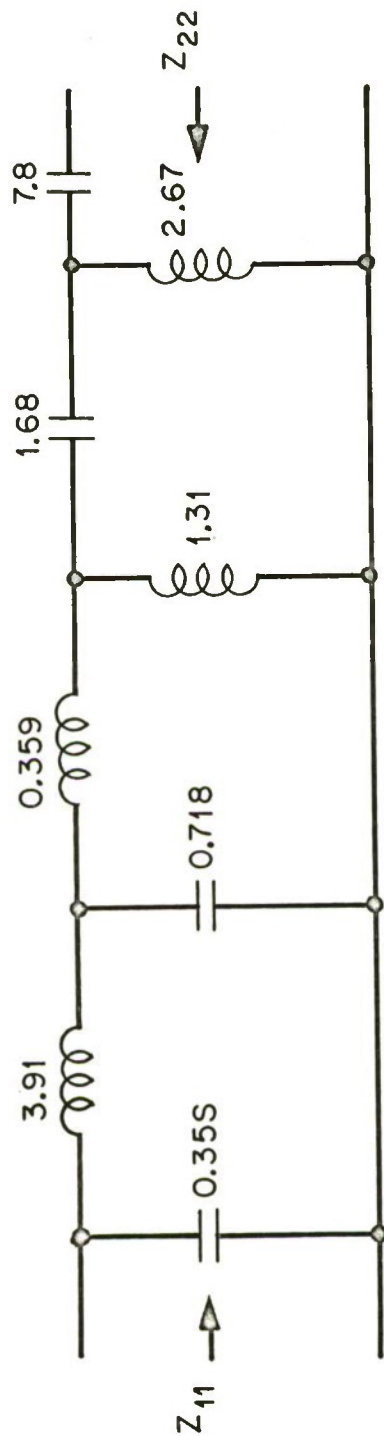FIG. 16 BAND PASS NETWORK OF EXAMPLE I (before impedance levelling)

LaVoche   Fig. GRAYZEL

A-12849-5
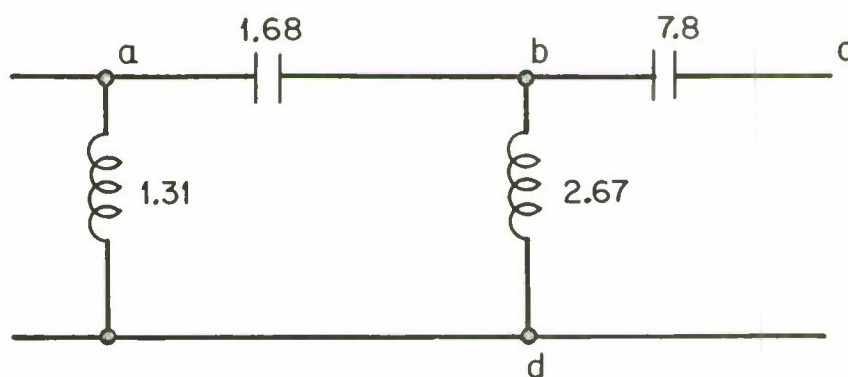
FIG. 17 ASSYMTOTIC BEHAVIOR OF NETWORK FIG. 16 AS $\omega \to 0$
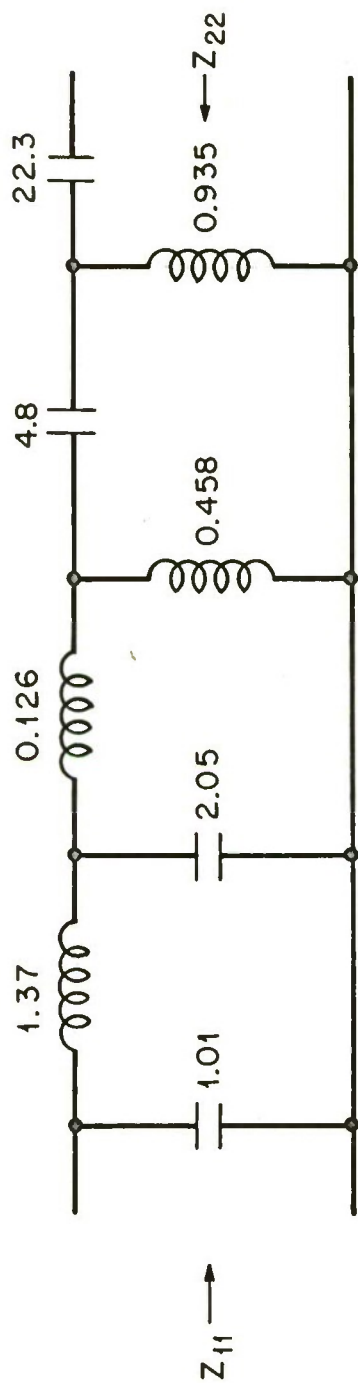
FIG.18 BAND PASS NETWORK OF EXAMPLE I

1-3-61 Lapierre Fig.

GRAYZEL

A-128649-17

Each network must now be impedance levelled to 8 ohms which involves multiplying each inductance by a factor of eight and each capacitance by 1/8. The frequency must also be scaled such that $\omega = 1$ corresponds to 8000 cycles. This involves multiplying each inductance and capacitance by 1/8000. Hence, combining these operations, each inductance is multiplied by 10^{-3} and each capacitance by $1/64 \times 10^{-3}$. The final network is shown in Figure 19.

VIII. SYNTHESIS PROCEDURE FOR L FILTERS

Eq. (48) can be rewritten in the form

$$|Z_{Ti}(j\omega)|^2 = \frac{L_n(\frac{\omega}{\omega_i})^2 - L_n(\frac{\omega}{\omega_{i+1}})^2}{[1 + L_n(\frac{\omega}{\omega_{i+1}})^2][1 + L_n(\frac{\omega}{\omega_i})^2]} \tag{76}$$

Since $L_n(\omega/\omega_i)$ is monotonic and satisfies Eq. (27), it is clear that the transmission zeros occur at zero, infinity and at complex frequencies corresponding to the roots of the numerator of Eq. (76). These complex zeros complicate the problem considerably and prevent a simple ladder synthesis of the corresponding network.

To synthesize the network corresponding to $|Z_{Ti}(j\omega)|^2$, $Z(s)$ must first be determined. The impedance $Z(s)$ can be found as follows:

Let

$$\text{Re}\,[Z'_i(s)]_{s=j\omega} = |Z'_{Ti}(j\omega)|^2 = \frac{1}{1 + L_n(\frac{\omega}{\omega_{i+1}})^2} \tag{77}$$
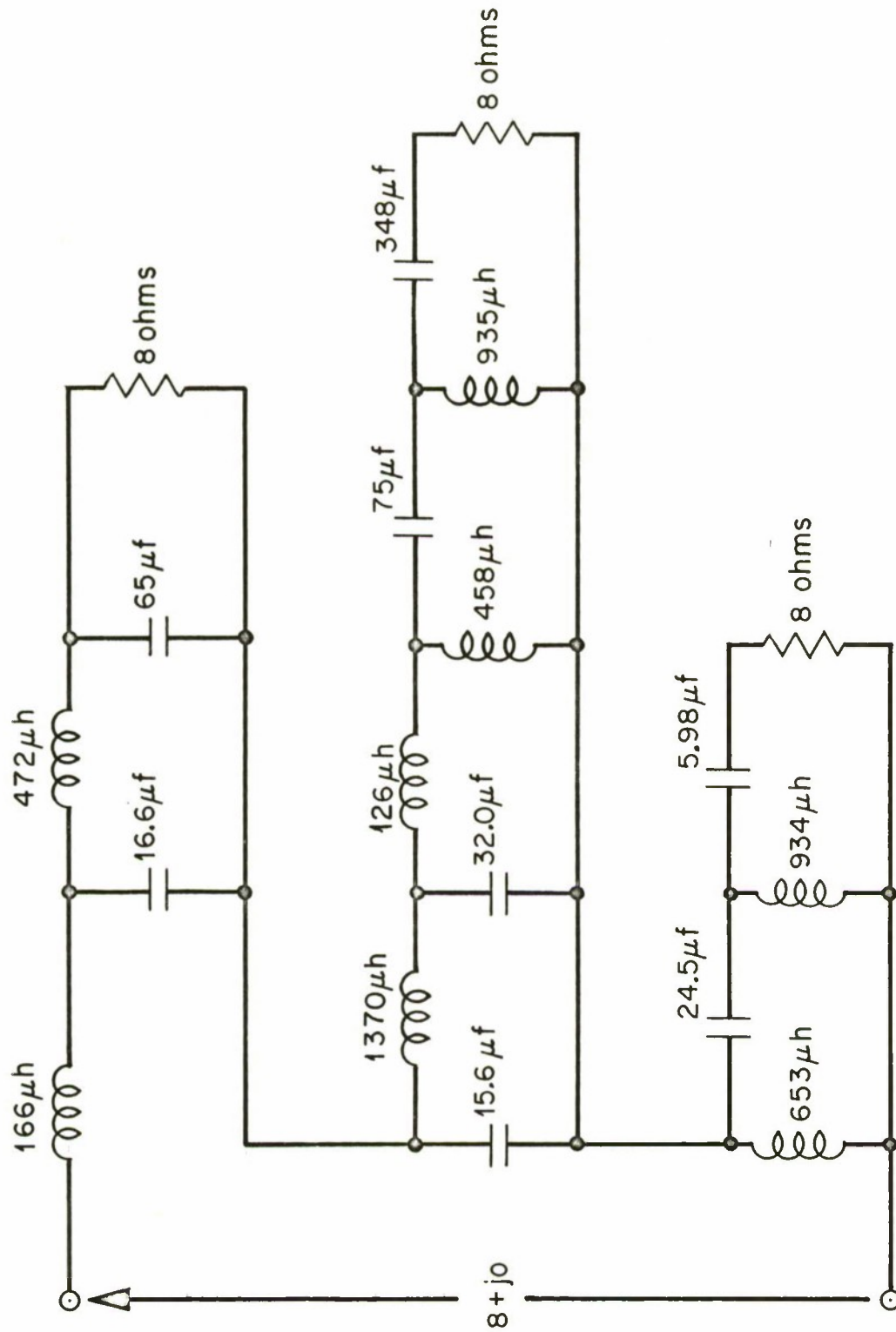
FIG. 19 BAND SEPARATION FILTER EXAMPLE I

and

$$Re[Z_i''(s)]_{s=j\omega} = |Z_{Ti}''(j\omega)|^2 = \frac{1}{1+L_n(\frac{\omega}{\omega_i})^2} \qquad (78)$$

Subtracting

$$Re[Z_i'(s)] - Re[Z_i''(s)] = |Z_{Ti}'(j\omega)|^2 - Z_{Ti}''(j\omega)|^2$$

$$= \frac{1}{1+L_n(\frac{\omega}{\omega_{i+1}})^2} - \frac{1}{1+L_n(\frac{\omega}{\omega_i})^2} \qquad (79)$$

Using Eq. (48)

$$Re[Z_i'(s) - Z_i''(s)] = |Z_{Ti}(j\omega)|^2 \qquad (80)$$

but

$$Re[Z_i(s)] = |Z_{Ti}(j\omega)|^2 \qquad (81)$$

Therefore, if

$Z_i(s)$, $Z_i'(s)$ and $Z_i''(s)$ are all minimum reactive, then

$$Z_i(s) = Z_i'(s) - Z_i''(s). \qquad (82)$$

Let us define

$$Z^O(s) = \frac{1}{1 + L_n(-s)^2} \qquad (83)$$

Then by Eqs. (77) and (78),

$$Z_i'(s) = Z^O\left(\frac{s}{\omega_{i+1}}\right) \qquad (84a)$$

$$Z_i''(s) = Z^O\left(\frac{s}{\omega_i}\right) \qquad (84b)$$

In Table IV, Z_i^O is tabulated for $n = 2, 3, \ldots, 6$. From this table using Eqs. (82) and (84), $Z_i(s)$ is easily determined.

Having determined $Z_i(s)$, the Darlington synthesis procedure is now employed. It should be pointed out that since $m_1 m_2 - n_1 n_2$ will not be a perfect square, augmentation is necessary. This increases the number of elements required, hence, a $(2n)^{th}$ order Butterworth may have the same number of elements as an nth order L filter. Hence, for the same number of elements, the Butterworth may yield a better characteristic. It should also be noted that it will, in general, be necessary to use coupled coils in the synthesis of the L filters, which is usually undesirable. These considerations lead the author to feel that the use of the Butterworth characteristic is more desirable.

n	$Z(s)$
3	$\dfrac{0.672s^2 + 0.822s + 0.577}{s^3 + 1.310s^2 + 1.356s + 0.577}$
4	$\dfrac{0.620s^3 + 0.969s^2 + 0.939s + 0.408}{s^4 + 1.563s^3 + 1.866s^2 + 1.241s + 0.408}$
5	$\dfrac{0.613s^4 + 0.950s^3 + 1.135s^2 + 0.705s + 0.224}{s^5 + 1.551s^4 + 2.203s^3 + 1.693s^2 + 0.898s + 0.224}$
6	$\dfrac{0.612s^5 + 1.056s^4 + 1.438s^3 + 1.132s^2 + 0.493s + 0.141}{s^6 + 1.726s^5 + 2.690s^4 + 2.433s^3 + 1.633s^2 + 0.680s + 0.141}$

Table IV

Input Impedance of L-Type Filter

IX.   APPLICATION TO TRANSMISSION LINE NETWORKS

Let us consider an input impedance $Z(s)$ and corresponding $|Z_T(j\omega)|^2$ which can be synthesized as a lossless ladder network terminated in a resistive load consisting of series and shunt lumped inductances and capacitances. It has been shown[6] that the input impedance $Z(\lambda)$ and corresponding $Z_T(j\Omega)$ can be synthesized in a ladder network using transmission line components where

$$\lambda = \tanh \frac{s}{4f_o} = \Gamma + j\Omega \qquad (85)$$

The elements used consist of series and shunt shorted and open stubs, all a quarter wavelength long at frequency f_o and sections of transmission line of this same length called unit elements.   The realization of the series stub in coaxial transmission line is discussed in Reference (7) while the realization in strip line is discussed in Reference (8).

Since the Butterworth characteristic yields band separation filters composed of ladder networks with series and shunt inductances and capacitances, it can be synthesized using transmission line components.   Since λ is a transformation of the complex frequency scale and Ω, a transformation of the ω axis, it follows from Eqs. (86) and (87)

$$\sum_{i=1}^{m} Z_{Ti}(j\Omega) = 1 \qquad (86)$$

$$\sum_{i=1}^{m} Z_i(\lambda) = 1 \qquad (87)$$

and, hence, the transmission line networks are complementary.

The frequency f_o is chosen as the largest frequency of interest, since the frequency f_o corresponds to λ equal to infinity. The filter characteristics that can be achieved can be determined from Figures 5, 6, 7 and 8 by substituting Ω for ω. To determine the characteristic as a function of frequency, the relation

$$\Omega = \tan \frac{\omega}{4f_o} \tag{88}$$

is then used.

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13. ABSTRACT

A band separation filter is a network with one input and m outputs, each corresponding to a different portion of the frequency spectrum. When a voltage is applied to the input terminal, it will appear at one of the output terminals only slightly attenuated. The filter considered here is a lossless network with each output terminal terminated in a one ohm resistance. The further condition that the input impedance of this network equals $1 + j0$ for all frequencies is imposed.

In this report a sufficient condition for realizability on the m transfer impedances is derived. It is shown that Butterworth characteristics for each of the m transfer impedances can be achieved with networks synthesizable in ladder form. It is also shown that L filter characteristics are also realizable but that the synthesis procedure is more complicated and necessitates coupled coils. Normalized curves of the attenuation characteristics for each type are presented.

The extension of this method to transmission line networks is discussed, and it is shown that the Butterworth characteristic can be achieved with this type of element.

14. KEY WORDS

bandpass filters	transmission line networks
Butterworth network design	band separation filter